CS230: Exercise sheet 1 - solutions

1. (a) $S_1, S_2 : \mathbb{P}\,\mathbb{N}$
   (b) $\{x : \mathbb{N} \mid (x \in S_1 \lor x \in S_2) \land \neg\,(x \in S_1 \land x \in S_2)\}$
   (c) $(S_1 \cup S_2) \setminus (S_1 \cap S_2)$
   (d) $(S_1 \setminus S_2) \cup (S_2 \setminus S_1)$

2. (a) $\mathbb{P}\,\mathbb{Z}$
   (b) $\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$
   (c) $(\mathbb{Z} \times \mathbb{Z}) \times \mathbb{P}(\mathbb{Z} \times \mathbb{Z})$
   (d) Incorrectly typed.
   (e) $\mathbb{P}\,\mathbb{P}(\mathbb{Z} \times \mathbb{Z})$
   (f) $(\mathbb{P}\,\mathbb{Z}) \times (\mathbb{P}?)$ - but we don't know what ? is.

3. (a) $\mathbb{P}(\mathbb{Z} \times \mathbb{P}\,\mathbb{Z})$
   (b)

$$
\begin{array}{|l}
R : \mathbb{P}(\mathbb{Z} \times \mathbb{P}\,\mathbb{Z}) \\
\hline
\forall\, x : \mathbb{Z};\ y : \mathbb{P}\,\mathbb{Z} \bullet \\
\qquad (x, y) \in R \Leftrightarrow (x \in 0 \mathinner{..} 4 \land y = 2 * (x \operatorname{\mathsf{div}} 2)) \\
\end{array}
$$

4.

$$
\begin{array}{|l}
[X] \\
\hline
\mathit{singleton} : \mathbb{P}\,\mathbb{P}\,X \\
\hline
\forall\, s : \mathbb{P}\,X \bullet \\
\qquad s \in \mathit{singleton} \Leftrightarrow s \subseteq X \land \#s = 1 \\
\end{array}
$$

   OR

$$\mathit{singleton}[X] == \{s : \mathbb{P}\,X \mid \#s = 1\}$$

   OR

$$\mathit{singleton}[X] == \{x : X \bullet \{x\}\}$$

5. (a) i. $\forall\, x, y : \mathbb{Z} \bullet x > y \Rightarrow x - y > 0$
   $\forall\, x, y : \mathbb{Z} \mid x > y \bullet x - y > 0$
   ii. $\forall\, x : \mathbb{N} \bullet x \neq 0 \Rightarrow (\exists\, y : \mathbb{N} \bullet y < x)$
   $\forall\, x : \mathbb{N} \mid x \neq 0 \bullet (\exists\, y : \mathbb{N} \bullet y < x)$
   iii. $\exists\, x : \mathbb{P}\,\mathbb{Z} \bullet 1 \in x \land -1 \notin x$
   $\exists\, x : \mathbb{P}\,\mathbb{Z} \mid 1 \in x \bullet -1 \notin x$
   iv. $\forall\, x : \mathbb{F}\,\mathbb{Z} \bullet \exists\, y : \mathbb{Z} \bullet y \in x \land (\forall\, z : \mathbb{Z} \bullet z \in x \Rightarrow y < z)$
   $\forall\, x : \mathbb{F}\,\mathbb{Z} \bullet \exists\, y : \mathbb{Z} \mid y \in x \bullet (\forall\, z : \mathbb{Z} \mid z \in x \bullet y < z)$

(b) i. $\{x : 1 \mathinner{..} 5 \bullet \{x\}\}$

ii. $\{x, y : S \mid x \neq y \bullet (x, y)\}$

iii. $\{x : \mathbb{N} \mid (\forall\, y, z : \mathbb{N} \mid y * z = x \bullet y = x \lor z = x)\}$

iv. $\{x : \mathbb{N} \bullet 3 * x\}$

6. (a) No. Some pairs, such as $(ad, 15)$ are not achievable scores.

(b)

$$\begin{array}{|l} scores : \mathbb{P}(A \times A) \\ \hline scores = \{m, n : A \mid m = ad \Rightarrow n = 40 \land \\ \qquad\qquad\qquad\qquad n = ad \Rightarrow m = 40 \land \\ \qquad\qquad\qquad\qquad \neg\, (m = g \land n = g)\} \end{array}$$

7. In a strongly typed language like Z, an operation will return a value of a fixed type. It cannot return a *VALUE* sometimes and a *REPORT* at others. To overcome this we could make a larger type which has branches constructed from both *VALUE* and *REPORT*.

$$OUTPUT ::= val\langle\langle VALUE \rangle\rangle \mid rep\langle\langle REPORT \rangle\rangle$$