CS230 Exercise sheet 1

Types and declarations

1. (a) Write a declaration for two subsets S_1 and S_2 of the natural numbers.
   (b) Write a set comprehension term to denote the set of all numbers which are in S_1 or S_2 but not in both.
   (c) Write a term denoting the same as the previous part using only the operations $\cup$, $\cap$ and $\setminus$.
   (d) Write a term denoting the same set but using only $\cup$ and $\setminus$.

2. What is the type of each of the following (assuming $\mathbb{Z}$ is the basic type for numbers)?
   (a) $\{0, 1\}$
   (b) $(1, 2, 3)$
   (c) $((0, 1), \{(0, 1), (1, 0)\})$
   (d) $\{\{0, 1\}, (1, 1)\}$
   (e) $\{\{(0, 1)\}\}$
   (f) $(\{1\}, \emptyset)$

3. Let R be:

   $$\{(0, \{0\}), (1, \{0\}), (2, \{2\}), (3, \{2\}), (4, \{4\})\}$$

   (a) What is the type of R?
   (b) How could it be defined in an axiomatic definition without enumerating all the members?

4. Suppose $singleton[X]$ is intended to be the set of all subsets of X with exactly one member. Give a generic definition for this.

5. (a) Write each of the following expressions in Z set notation.
      i. For integers x and y with x greater than y, the difference $x - y$ is positive.
      ii. For every member of $\mathbb{N}$ other than 0 there are members of $\mathbb{N}$ which are smaller.
      iii. There is a set of integers which contains 1 but not -1.
      iv. Every finite set of integers has a least element.

      Z also uses an extended notation for quantified expressions. We'll be covering this soon - at which point, try writing the above using the extended notation.
   (b) Z also allows an extended set notation. Write set expressions for the following, making use of the extended notation in order to make the expressions as simple as possible.

i. The set containing the singleton sets of each of the numbers 1 to 5 inclusive.
ii. The set of pairs of distinct elements of a set S.
iii. The set of all prime numbers.
iv. The set of all members of $\mathbb{N}$ which are multiples of 3.

6. This is an extra question for anyone who knows the rules of tennis. Let $A = \{0, 15, 30, 40, ad, g\}$ denote the set of scores a player can achieve in a game of tennis.

   (a) Is the set of scores achievable by a pair of opponents during a game equal to the Cartesian product $A \times A$? Why?

   (b) Using an axiomatic definition, define the set of scores achievable by a pair of players during the game.

7. A Z operation has been defined to return a result of type $VALUE$. However, if an error occurs, a report of a different type $REPORT$ should be returned instead. What is the problem with specifying this? Give a free type definition that could help.